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Isotopic dependence of fusion cross-sections – linear relationships

Rajeev K. Puri, Manoj K. Sharma, Raj K. Gupta

Centre of Advanced Study in Physics, Panjab University, Chandigarh 160014, India

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Abstract. A systematic study of isotopic dependence of fusion cross-section is carried out by adding neutrons gradually to N=Z colliding nuclei. We find that fusion barrier position increases and height decreases, both linearly with the increase of N/Z ratio of the compound system. The increase in barrier position is larger compared to decrease in barrier height. In terms of these linear relationships, a parameterized form of fusion cross-sections is given for the neutron-rich colliding nuclei. The fusion cross-sections are also enhanced linearly with the N/Z ratio, and this enhancement is larger for lower incident centre-of-mass energies and independent of the choice of reaction partners. Experimental data and other theoretical studies are called for to verify these results.

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In recent years, due to the availability of radioactive, neutron-rich beams, it has become possible to synthesize new and neutron-rich heavy nuclei [1,2]. The fusion of neutron-rich nuclei result in nuclei farther away from the $\beta-$stability nuclei. In addition to the important nuclear structure information [3,4], like the "thick neutron-skin" (the neutron-halo), subtle fusion characteristics have also started to emerge. The fusion cross-sections are found to get enhanced considerably for the neutron-rich projectiles compared to nuclei lying near the $\beta-$stability line [1,2]. Such an enhancement of fusion cross-section is considered [1] due to the lowering of fusion barrier heights V_B for neutron-rich colliding nuclei, though fusion barrier positions R_B are known to play an equally important role in fusion studies at low energies [5]. This is further illustrated in Fig.1 where measured fusion cross-sections σ_{fus} and the reduced cross-sections σ_{fus}/R_B^2 for a few systems are plotted as a function of reduced center-of-mass energy E_{cm}/V_B. Then, there are other theoretical questions [6-11], like the halo nature of projectile, direct breakup of projectile versus soft dipole resonance between the core and halo neutrons, flow of neutrons during the collision process, cold synthesis and cluster decay of nuclei at the neutron-drip line, etc., which are not yet established experimentally [2,12,13]. Also, there is no systematic study available so far, either experimental or theoretical, on the isotopic dependence of fusion cross-section due to adding neutrons to, say, N=Z colliding nuclei. Such a study can be very useful for cold synthesis of new and superheavy elements [1,11].

In this Letter, we study the above noted effect of adding neutrons to N=Z colliding nuclei on fusion barriers (both heights and positions) and hence the fusion excitation functions. For this purpose, we use the Skyrme energy density formalism (SEDF) which has been found quite successful in explaining the fusion of two colliding nuclei at low energies [14-17]. We have chosen the colliding nuclei from (1s-0d) to 0g shells where SEDF is found to work nicely and even analytical solutions are possible [17]. Specifically, we have studied fifteen reactions involving different isotopes of $_{20}Ca$ and $_{28}Ni$. We begin with N=Z compound systems and go upto N=2Z compound systems by adding neutrons gradually. Only the even-even nuclei are considered.

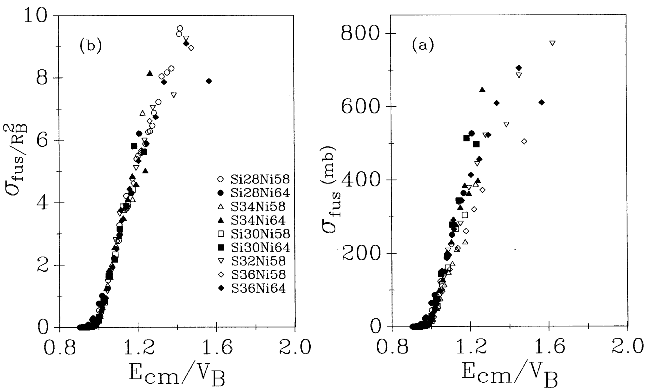

Fig. 1. **a** Measured fusion cross-sections σ_{fus} (mb), **b** the reduced cross-sections σ_{fus}/R_B^2, as function of the reduced centre-of-mass energy E_{cm}/V_B for some colliding systems

In SEDF, the nucleus-nucleus interaction potential is calculated as a difference of the energy expectation $E = \int H(\boldsymbol{r})d\boldsymbol{r}$ of colliding nuclei at a separation distance R and at infinity [17]:

$$\begin{aligned} V_N(R) &= E(R) - E(\infty) \\ &= \int \big\{H(\rho,\tau,\boldsymbol{J}) - H_1(\rho_1,\tau_1,\boldsymbol{J_1}) \\ &\quad -H_2(\rho_2,\tau_2,\boldsymbol{J_2})\big\} d\boldsymbol{r}. \end{aligned} \tag{1}$$

Here, $H(\boldsymbol{r})$ is the Hamiltonian density [18], defined in terms of nucleon, kinetic energy and spin densities, ρ_i, τ_i, $\boldsymbol{J}_i$, respectively, for even-even spherical nuclei as

$$\begin{aligned} &H(\rho,\tau,\boldsymbol{J}) = \\ &\frac{\hbar^2}{2m}\tau + \frac{1}{2}t_0[(1+\frac{1}{2}x_0)\rho^2 - (x_0+\frac{1}{2})(\rho_n^2+\rho_p^2)] \\ &+\frac{1}{4}(t_1+t_2)\rho\tau + \frac{1}{8}(t_2-t_1)(\rho_n\tau_n+\rho_p\tau_p) \\ &+\frac{1}{16}(t_2-3t_1)\rho\nabla^2\rho + \frac{1}{32}(3t_1+t_2)(\rho_n\nabla^2\rho_n+\rho_p\nabla^2\rho_p) \\ &+\frac{1}{4}t_3\rho_n\rho_p\rho - \frac{1}{2}W_0(\rho\boldsymbol{\nabla}\cdot\boldsymbol{J}+\rho_n\boldsymbol{\nabla}\cdot\boldsymbol{J}_n+\rho_p\boldsymbol{\nabla}\cdot\boldsymbol{J}_p). \end{aligned} \tag{2}$$

The terms containing $\boldsymbol{J^2}$ are neglected. The subscripts n and p refer to neutrons and protons, respectively, and the Skyrme force parameters, chosen for force SIII, are $x_0 = 0.45$, $t_0 = -1128.75\ MeV fm^3$, $t_1 = 395.00\ MeV fm^5$, $t_2 = -95.00\ MeV fm^5$, $t_3 = 14000\ MeV fm^6$ and $W_0 = 120\ MeV fm^5$. Also, $\rho_i = \rho_{n_i} + \rho_{p_i}$, $\tau_i = \tau_{n_i} + \tau_{p_i}$ and $\boldsymbol{J}_i = \boldsymbol{J}_{n_i} + \boldsymbol{J}_{p_i}$, and, using sudden approximation, $\rho = \rho_1 + \rho_2$, $\tau = \tau_1 + \tau_2$ and $\boldsymbol{J} = \boldsymbol{J}_1 + \boldsymbol{J}_2$. The nucleonic density in present study are taken from two parameter Fermi density with constants taken from [17]. For neutron rich nuclei, the corresponding density of same mass nucleus given in [17] is used. In other words, for example, the density parameter of ^{60}Ca are the same as that of ^{60}Zn. Apparently, by doing so we neglect the explicit contribution due to additional skin of neutrons in neutron rich nuclei. Since spin-dependent and spin-independent terms in (2) occur separately, (1) can be separated as

$$V_N(R) = V_P(R) + V_J(R), \tag{3}$$

with

$$V_P(R) = \int [H(\rho,\tau) - H_1(\rho_1,\tau_1) - H_2(\rho_2,\tau_2)]\, d\boldsymbol{r}, \tag{4}$$

and

$$V_J(R) = \int [H(\rho,\boldsymbol{J}) - H_1(\rho_1,\boldsymbol{J}_1) - H_2(\rho_2,\boldsymbol{J}_2)]\, d\boldsymbol{r}. \tag{5}$$

The spin-independent part is calculated within the standard proximity theorem and spin-dependent part is further divided into closed j-shell core and the valence particles or holes [19]. Further details can be seen in [17, 19] and earlier references therein.

Adding Coulomb interaction to $V_N(R)$, we get the total interaction potential

$$V(R) = V_N(R) + \frac{Z_1Z_2e^2}{R}, \tag{6}$$

Table 1. Fusion barrier positions R_B and heights V_B for different colliding nuclei using Skyrme force SIII ($\lambda = 0$)

System	N/Z Ratio	Exact R_B	Exact V_B
$^{40}Ca + ^{40}Ca$	1.000	9.68	54.26
$^{40}Ca + ^{48}Ca$	1.200	10.06	52.60
$^{40}Ca + ^{60}Ca$	1.500	10.61	50.33
$^{48}Ca + ^{60}Ca$	1.700	10.94	48.92
$^{60}Ca + ^{60}Ca$	2.000	11.43	46.98
$^{40}Ca + ^{56}Ni$	1.000	9.95	73.72
$^{40}Ca + ^{58}Ni$	1.042	10.07	73.16
$^{40}Ca + ^{60}Ni$	1.083	10.16	72.77
$^{40}Ca + ^{62}Ni$	1.125	10.19	72.32
$^{56}Ni + ^{56}Ni$	1.000	10.02	100.59
$^{62}Ni + ^{62}Ni$	1.214	10.65	96.70
$^{68}Ni + ^{68}Ni$	1.429	11.11	93.76
$^{74}Ni + ^{74}Ni$	1.643	11.39	91.66
$^{80}Ni + ^{80}Ni$	1.857	11.72	89.49
$^{86}Ni + ^{86}Ni$	2.071	12.08	87.21

whose barrier position R_B and height V_B are determined by the conditions

$$\left[\frac{dV(R)}{dR}\right]_{R=R_B} = 0, \text{ and } \left[\frac{d^2V(R)}{dR^2}\right]_{R=R_B} \le 0. \tag{7}$$

This allows us to calculate the fusion cross-sections (in mb), using the sharp cut-off model,

$$\sigma_{fus}(mb) = 10\pi R_B^2 \left(1 - \frac{V_B}{E_{cm}}\right). \tag{8}$$

Table 1 gives the calculated fusion barrier positions R_B and heights V_B, along with the N/Z ratios for the compound systems ($N = N_1 + N_2$, $Z = Z_1 + Z_2$; N_i and Z_i being the neutron and proton numbers of the two colliding nuclei). In each group of reactions, we begin with N/Z = 1 and increase this ratio upto ≈ 2 by adding neutrons to one (or both) the reaction partner(s). We notice from Table 1 that the addition of neutrons has two effects: (i) The barrier height is lowered, and (ii) the barrier position is increased. In order to systematise this result, we define the percentage decrease and increase of V_B and R_B, respectively, as the decrease/increase over the N = Z case:

$$\Delta R_B(\%) = \frac{R_B - R_B^0}{R_B^0} \times 100 \tag{9}$$

$$\Delta V_B(\%) = \frac{V_B - V_B^0}{V_B^0} \times 100 \tag{10}$$

where $R_B^0 = R_B(N = Z), V_B^0 = V_B(N = Z)$, the barrier positions and heights for the N = Z cases. The ΔR_B and ΔV_B are calculated for the three groups of reactions listed in Table 1 and the results of this calculation are plotted in Figs. 2(a) and (b). It is interesting to find that both

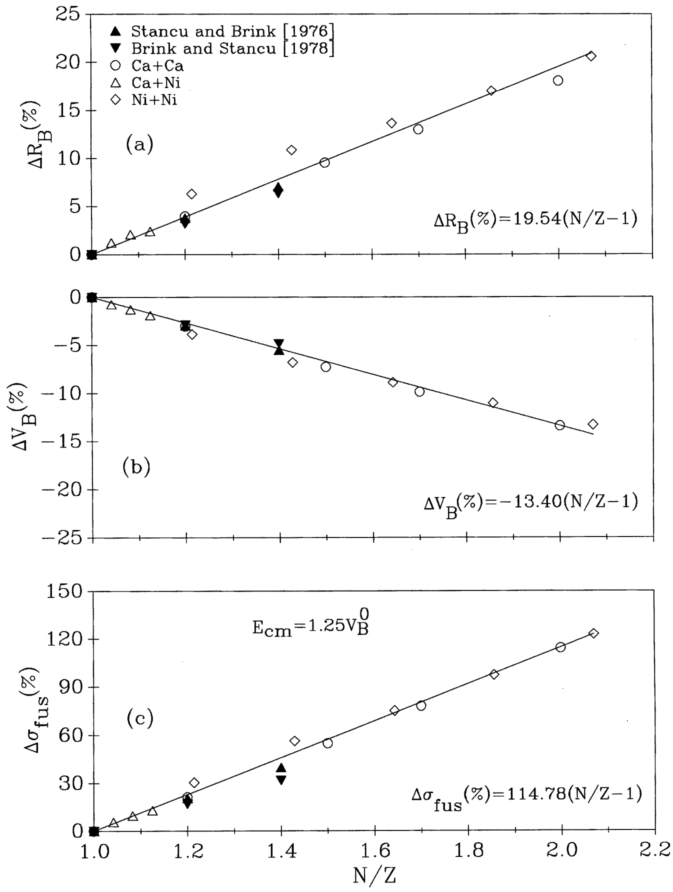

Fig. 2. **a** Percentage increase of barrier position R_B **b** percentage decrease of barrier height V_B, over their R_B and V_B values for the N = Z case, as a function of the N/Z ratios of the compound systems. **c** the same as above for the fusion cross-section σ_{fus}. In each case, the open symbols are the results of our calculations for three groups of reactions under study, the solid symbols are the earlier calculations and the lines show the straight line fits

ΔR_B and ΔV_B, respectively, increase and decrease monotonically with N/Z ratios of the compound systems and are represented by simple straight line equations

$$\Delta R_B = 0.1954(N/Z-1) \tag{11}$$

$$\Delta V_B = -0.1340(N/Z-1). \tag{12}$$

In Fig. 2 are also plotted, the results of other model calculations [14], which also lie exactly on the above two lines. Notice that the increase in R_B is rather stronger than the decrease in V_B ($\sim$ 20% is to be compared with $\sim$ 13% for the N = 2Z case). This is important, particularly because R_B occurs as squared in (8) for fusion cross-sections. The decrease of barrier height V_B can be understood as the increase of attractive nuclear interactions due to the addition of neutrons, with the repulsive Coulomb barrier remaining same.

(11) and (12) allow us to calculate the fusion cross-sections for all N/Z >1 compound systems in terms of the

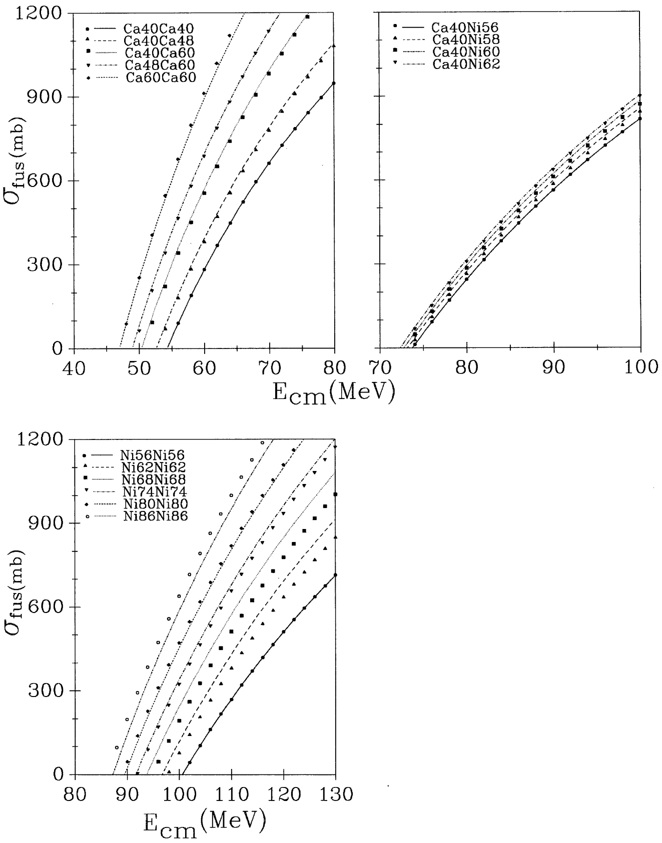

Fig. 3. Exact fusion cross-sections σ_{fus} (lines) calculated by using (8) and the parametrically fitted cross-sections σ_{fus}^{fit} (solid symbols) using (13), as a function of the centre-of-mass energy E_{cm}

cross-section for the N = Z case, as

$$\sigma_{fus}^{fit}(mb) = 10\pi R_B^{0^2}(1+\Delta R_B)^2\left(1-\frac{V_B^0(1+\Delta V_B)}{E_{cm}}\right). \tag{13}$$

Thus, knowing the barrier position and height for the N = Z case, we can calculate the fusion cross-sections for all neutron-rich colliding nuclei. This is depicted in Fig. 3 (solid/open symbols) for all the three groups of reactions under study. For comparisons, we have also shown here the exact excitation functions (lines) calculated by using in (8) the actual R_B and V_B from Table 1. The results of two expressions (the exact (8) and the fitted or parametrized (13)) are almost identical. An interesting result of Fig. 3 (symbols or lines) is that fusion cross-sections get strongly enhanced for isotopes with very neutron-rich configurations.

Defining the (percentage) change in fusion cross-sections, similar to (9) and (10), as

$$\Delta\sigma_{fus}(\%) = \frac{\sigma_{fus}(E_{cm}^0)-\sigma_{fus}^0(E_{cm}^0)}{\sigma_{fus}^0(E_{cm}^0)}\times 100, \tag{14}$$

we find that $\Delta\sigma_{fus}$ is also a monotonically increasing function of N/Z ratio at each value of the reduced $E^0_{cm} = E_{cm}/V^0_B$, (see Fig. 2(c)), and is given by the straight line equation

$$\Delta\sigma_{fus} = C(E^0_{cm})(N/Z - 1), \tag{15}$$

with the constant C = 1.872, 1.148, 0.906, 0.785, 0.713, 0.664, 0.630 and 0.604, respectively, at $E^0_{cm} = 1.125$, 1.250, 1.375, 1.500, 1.625, 1.750, 1.875 and 2.000, a gradually decreasing function of reduced centre-of-mass energy. Note that (14) is based on (8) and, σ^0_{fus} is the fusion cross-section for N = Z case. We have also plotted in Fig. 2(c) the results of other model calculations [14]. The interesting result is that the change $\Delta\sigma_{fus}$ depends on N/Z ratio and the centre-of-mass energy and not on the nature of reaction partners. In other words, $\Delta\sigma_{fus}$ increases as N/Z ratio increases and decreases as E_{cm} increases but is same for all the three groups of reactions. We have also analyzed the effect of increasing the surface thickness on fusion barriers and fusion cross-sections. For ^{40}Ca-^{40}Ca colliding nuclei, an increase in the surface thickness (of each Ca- nucleus) by a factor of two, results in ∼ 28% increase in barrier position and ∼ 25% decrease in barrier height which would apparently enhance the fusion cross-section further (depending on incident center of mass energy). This point is being further analyzed.

Summarizing, we have carried out a systematic study of fusion barriers and cross-sections for neutron-rich colliding nuclei by starting from the N = Z compound system and adding neutrons gradually up to N = 2Z. We find that addition of neutrons modifies both the position and height of the barrier —- the position increases and height decreases linearly with the N/Z ratio of the compound system. These linear relationships allow us to write a simple parametrized form of the fusion cross-sections for neutron-rich colliding nuclei in terms of the barrier position and height of the N=Z compound system. The fusion cross-sections for neutron-rich colliding nuclei are found greatly enhanced, with the cross-section increasing with the increase of N/Z ratio but decreasing with the increase of incident center-of -mass energy. However, the change in fusion cross-section is independent of the choice of reaction partners —— a result which needs an immediate experimental verification.

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